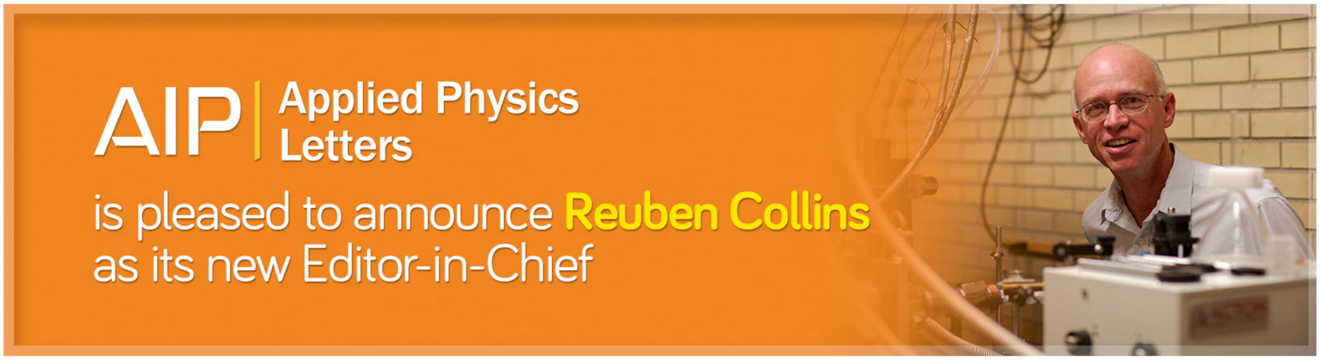
AIP | The Journal of Chemical Physics

A new definition of atomic charges based on a variational principle for the electrostatic potential energy

TaiSung Lee, Darrin M. York, and Weitao Yang

Citation: The Journal of Chemical Physics **102**, 7549 (1995); doi: 10.1063/1.469086
View online: http://dx.doi.org/10.1063/1.469086
View Table of Contents: http://scitation.aip.org/content/aip/journal/jcp/102/19?ver=pdfcov
Published by the AIP Publishing



A new definition of atomic charges based on a variational principle for the electrostatic potential energy

Tai-Sung Lee
Department of Chemistry, Duke University, Durham, North Carolina 27708-0354

Darrin M. York
Department of Chemistry, Duke University, Durham, North Carolina 27708-0354 and North Carolina Supercomputing Center, MCNC, Research Triangle Park, North Carolina 27709-12889

Weitao Yang[a)]
Department of Chemistry, Duke University, Durham, North Carolina 27708-0354

(Received 27 May 1994; accepted 13 February 1995)

A unique definition of atomic charges in molecules is presented based on a variational principle involving the electrostatic potential energy. The method requires only the electron density as input, and does not rely on an arbitrary set of fitting points as do conventional electrostatic potential fitting procedures. The dipole moments and electrostatic potentials calculated from atomic charges obtained from this method agree well with those from self-consistent-field calculations. The new method also provides a spherical-atom potential model that may be useful in future generation molecular simulation force fields. © *1995 American Institute of Physics.*

INTRODUCTION

The concept of net atomic charges in molecules is fundamental to chemistry. For instance, atomic charge has been used to describe the processes of electronegativity equalization and charge transfer in chemical reactions,[1,2] and to model the electrostatic potential outside molecular surfaces.[3–5]

Atomic charges have been defined based on population analysis,[6–9] partitioning of the molecular density into atomic densities,[10,11] electronegativity equalization techniques,[12] electrostatic potential fitting procedures,[13,14] and generalized atomic polar tensors.[15] However, no single definition of atomic charge has universal applicability.[16] Consequently, one must choose a definition of atomic charge that is best suited to represent a given set of properties.

Frequently atomic charges are derived from experimental data or electronic structure calculations for the purpose of providing an empirical representation of the electrostatic field used in molecular simulations. The most commonly employed methods for deriving charges for this purpose involve least-squares fitting of the electrostatic potential at a given set of points.[13,14] Typically, points are chosen to lie in the region outside the van der Waals surface of the molecule, where interactions with other molecules are most significant. For small molecules this procedure generally gives relatively stable and intuitively reasonable results; however, in cases where the (χ^2) merit function is insensitive to certain charge variations, conventional fitting procedures have difficulties.[17] In these cases, the electrostatic potential derived (ESPD) charges depend on the choice of fitting points, and the atomic charge definition becomes ambiguous. Recently, methods have been developed that attempt to improve this situation by employing more robust merit functions.[17,18]

In this paper, we present a unique definition of atomic charge based on a variational principle involving the electrostatic potential energy. The method depends only on the molecular electron density, and does not require fitting points. The definition is based on the construction of spherical-atom model potentials, and hence is ideal for use in molecular simulations.

THEORY

Here we develop a definition of atomic charge based on the optimal representation of the molecular density by a superposition of spherical atomic densities. The atomic charge is determined from the corresponding spherical-atom model density by quadrature.

Given an electron density ρ, consider the quantity U as a functional of the trial potential $\tilde{\phi}$,

$$U(\tilde{\phi}) = \frac{1}{8\pi} \int \tilde{\phi}(\mathbf{r}) \nabla^2 \tilde{\phi}(\mathbf{r}) d\mathbf{r} + \int \tilde{\phi}(\mathbf{r}) \rho(\mathbf{r}) d\mathbf{r}. \qquad (1)$$

A necessary condition for U to take on a maximum value with respect to variations in the trial potential is[19,20]

$$\frac{\delta U}{\delta \tilde{\phi}(\mathbf{r})} = \frac{1}{4\pi} \nabla^2 \tilde{\phi}(\mathbf{r}) + \rho(\mathbf{r}) = 0. \qquad (2)$$

Hence, the trial potential $\tilde{\phi}$ that maximizes U is a solution of the Poisson equation for the input density ρ. This principle has been used by Luty *et al.*[21] to solve the Poisson equation.

It can be shown that maximizing U is equivalent to minimizing the Dirichlet functional F defined as

TABLE I. Exponents used in the Slater-type basis functions.[a]

Atom	H	Li	B	C	N	O	F	Cl
Exponent λ_A	2.66	2.07	3.58	4.27	4.91	5.64	6.29	1.71

[a] Unit: a.u.

[a)] Alfred P. Sloan Research Fellow.

0021-9606/95/102(19)/7549/8/$6.00

$$F=\int [\mathbf{E}(\mathbf{r})-\tilde{\mathbf{E}}(\mathbf{r})]^2 d\mathbf{r},$$

where $\mathbf{E}(\mathbf{r})$ and $\tilde{\mathbf{E}}(\mathbf{r})$ are the input and model electric fields, respectively. Since it is the electric field, not the potential, that determine the electrostatic force on an atom, this interpretation of the atomic charge definition may have particular relevance for molecular dynamic simulations. This principle has been previously employed to model electron densities using cone functions, Gaussian functions, and atomic orbitals.[22,23]

Here we model the potential by a set of spherical functions centered at the nuclear positions. Consider the trial potential ϕ of the form,

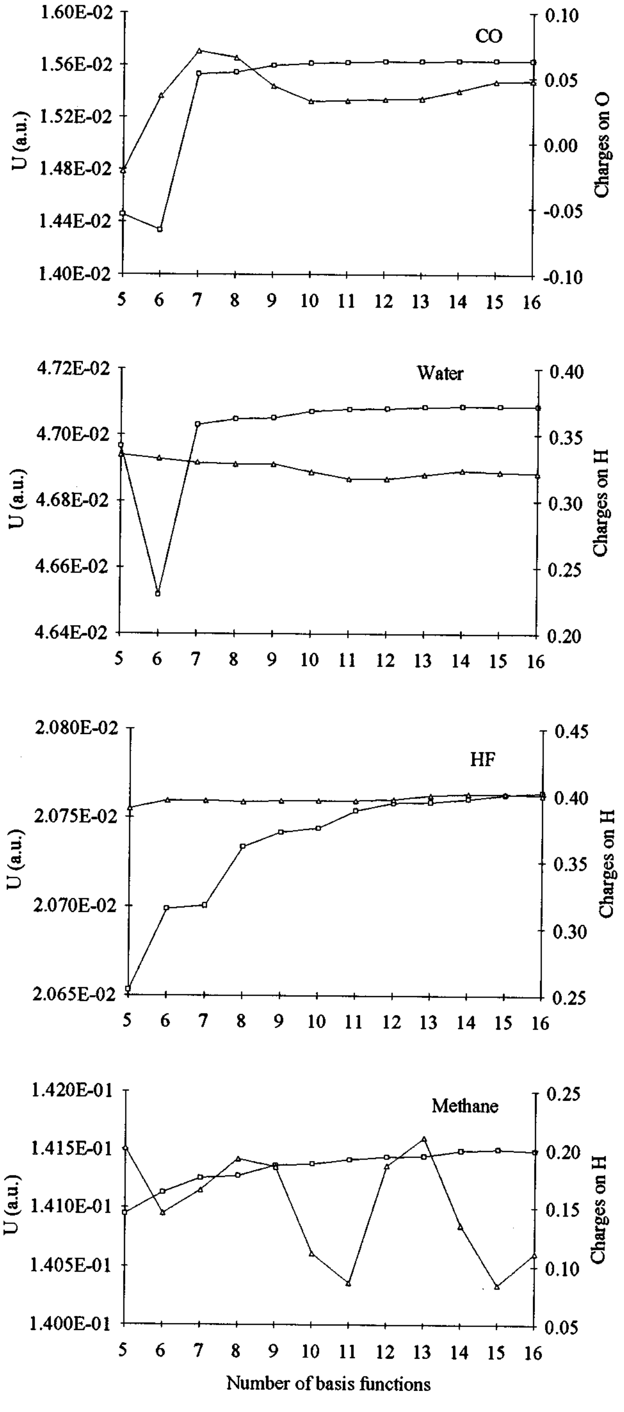


FIG. 1. The results of atomic charges (△) and U (□) with respect to number of basis functions.

TABLE II. Atomic charges obtained from fitting to the electrostatic potential (ESPD), from the present method using the deformation density (PM), and from the present method using the total density (PMTD).[a]

Molecule	Atom	ESPD	PM	PMTD
BH	B	−0.195	−0.266	−0.266
LiH	Li	0.645	0.721	0.721
LiF	Li	0.773	0.797	0.797
CO	C	−0.029	−0.047	−0.047
CO_2	C	0.669	0.694	0.696
H_2O	H	0.332	0.322	0.323
HF	H	0.403	0.401	0.402
HCl	H	0.190	0.155	0.160
NH_3	N	−0.879	−0.827	−0.828
CH_4	C	−0.644	−0.338	−0.362
Benzene	C	−0.118	−0.155	−0.156
CH_3F	C	−0.064	−0.177	−0.173
	H	0.085	0.114	0.113
	F	−0.193	−0.165	−0.166
HCOOH	H	0.097	0.103	0.104
	C	0.429	0.429	0.429
	O(carbonyl)	−0.440	−0.446	−0.447
	O	−0.500	−0.513	−0.514
	H	0.415	0.429	0.428
CH_3OH	C	−0.237	−0.475	−0.482
	H(*gauche*)	0.102	0.161	0.163
	H(*trans*)	0.185	0.262	0.265
	O	−0.521	−0.485	0.495
	H	0.368	0.376	0.377
NH_2CHO	N	−0.773	−0.655	−0.660
	H(*trans*)	0.346	0.300	0.301
	H(*cis*)	0.386	0.359	0.361
	C	0.487	0.372	0.375
	O	−0.471	−0.446	−0.447
	H(carbon)	0.026	0.070	0.069

[a] Unit: a.u.

TABLE III. Dipole moments derived from the ESPD charges, charges from present method (PM), the self-consistent-field density, and experimental data.[a]

Molecule	ESPD	PM	SCF density	Experiment
BH	0.453	0.618	0.615	0.51[b]
LiH	1.94	2.18	2.18	2.32[b]
LiF	2.31	2.38	2.41	2.49[b]
CO	0.062	0.101	0.094	0.043[b]
H_2O	0.740	0.719	0.714	0.727[b]
HF	0.698	0.695	0.691	0.719[b]
HCl	0.458	0.373	0.380	0.436[b]
NH_3	0.611	0.574	0.576	0.579[b]
CH_3F	0.668	0.654	0.657	0.727[b]
HCOOH	0.584	0.592	0.589	0.55[b]
CH_3OH	0.646	0.626	0.639	0.670[c]
NH_2CHO	1.42	1.41	1.42	1.47[c]

[a] Unit: atomic unit. For dipole moment, 1 atomic unit=2.54 D.

[b] A. A. Radzig and B. M. Smirnov, *Reference Data of Atoms, Molecules, and Ions* (Springer, Berlin, 1985).

[c] F. A. Momany, J. Phys. Chem. **82**, 592 (1978).

TABLE IV. ESPD charges obtained from different sets of fitting points (units, a.u.).

	CH_3F			CO		HCl	
Fitting point set	F	C	H	C	O	H	Cl
Set 1[a]	−0.1926	−0.0635	0.0854	−0.0290	0.0290	0.1900	−0.1900
Set 2[b]	−0.1915	−0.0645	0.0853	−0.0304	0.0304	0.1925	−0.1925
1.2 R^{c}_{VDW}	−0.1904	−0.0618	0.0841	−0.0276	0.0276	0.1951	−0.1951
2.0 R_{VDW}	−0.2084	0.0110	0.0658	−0.0285	0.0285	0.1797	−0.1797
2.0 R_{VDW}+6.0	−0.2602	0.2770	−0.0056	−0.0395	0.0395	0.1640	−0.1640
2.0 R_{VDW}+12.0	−0.3198	0.5691	−0.0831	−0.0420	0.0420	0.1607	−0.1607

[a] Set 1 is the set of concentric surfaces obtained from the scaled van der Waals surface (scaling factor 1.2, 1.4, 1.6, and 1.8).

[b] Set 2 is the set of concentric surfaces obtained from the scaled van der Waals surface (scaling factor 1.4, 1.6, 1.8, and 2.0).

[c] R_{VDW} stands for scaled van der Waals radius.

$$\tilde{\phi}(\mathbf{r})=\sum_{A,n} C_n^A f_n^A(|\mathbf{r}-\mathbf{R}_A|), \tag{3}$$

where the index A sums over different atoms and n sums over spherical functions $f_n^A(r)$ centered at the nuclear position $\mathbf{R}_A$. In this basis, the functional U can be rewritten

$$U=\frac{1}{8\pi}\sum_{A,n}\sum_{A',n'} C_n^A C_{n'}^{A'}\langle f_n^A|\nabla^2|f_{n'}^{A'}\rangle+\sum_{A,n} C_n^A\langle f_n^A|\rho\rangle. \tag{4}$$

Maximization of Eq. (4) under the normalization constraint

$$\int -\frac{1}{4\pi}\nabla^2\tilde{\phi}(\mathbf{r})d\mathbf{r}\equiv\int\tilde{\rho}(\mathbf{r})d\mathbf{r}=N, \tag{5}$$

where $\tilde{\rho}$ is the model density associated with the trial potential $\tilde{\phi}$ and N is the total number of electrons, leads to the matrix equation,

$$\mathsf{H}\mathbf{C}+\boldsymbol{\rho}-\gamma\mathbf{D}=0,$$

where

$$(\mathbf{C})_{A,n}\equiv C_n^A,$$

$$(\mathbf{H})_{A,n,A',n'}\equiv\frac{1}{4\pi}\langle f_n^A|\nabla^2|f_{n'}^{A'}\rangle,$$

$$(\rho)_{A,n}\equiv\langle f_n^A|\rho\rangle, \tag{6}$$

$$(\mathbf{D})_{A,n}\equiv-\frac{1}{4\pi}\int\nabla^2 f_n^A d\mathbf{r},$$

and γ is the Lagrange multiplier for the normalization constraint.

TABLE V. The variances of point charge model potential and spherical model potential, i.e., $\Sigma_i[(\tilde{\phi}_i-\phi_i)^2/n]$, where $\tilde{\phi}$ is the model potential and ϕ is the SCF electrostatic potential. The point sets are the same sets as Table IV.[a]

		CO_2	
Point set	ESPD point charge model	point charge model from presented method	Spherical model
Set 1	1.3×10^{-6}	1.4×10^{-6}	3.4×10^{-7}
Set 2	2.3×10^{-7}	1.9×10^{-7}	1.5×10^{-7}
1.2 R_{VDW}	6.2×10^{-6}	7.2×10^{-6}	1.0×10^{-6}
2.0 R_{VDW}	2.3×10^{-8}	2.3×10^{-8}	2.7×10^{-8}
2.0 R_{VDW}+6.0	5.3×10^{-11}	5.8×10^{-11}	5.8×10^{-11}
2.0 R_{VDW}+12.0	1.4×10^{-10}	4.4×10^{-11}	4.4×10^{-11}

[a] Unit: a.u.

The solution of the coefficient vector $\mathbf{C}$ is given by

$$\mathbf{C}=-\mathsf{H}^{-1}\boldsymbol{\rho}+\gamma\mathsf{H}^{-1}\mathbf{D}, \tag{7}$$

where γ is determined by

$$\gamma=\frac{N+\mathbf{D}^T\mathsf{H}^{-1}\boldsymbol{\rho}}{\mathbf{D}^T\mathsf{H}^{-1}\mathbf{D}}. \tag{8}$$

Solution of Eq. (7) and Eq. (8) gives the spherical model potential $\tilde{\phi}$ via Eq. (3). The model density $\tilde{\rho}$ is determined from the potential via Poisson's equation, and the atomic contributions of the model density and potential are given by

$$\tilde{\phi}_A(\mathbf{r})=\sum_n C_n^A f_n^A(|\mathbf{r}-\mathbf{R}_A|),$$

$$\tilde{\rho}_A(\mathbf{r})=-\frac{1}{4\pi}\nabla^2\tilde{\phi}_A(\mathbf{r}). \tag{9}$$

The atomic charge q_A is then determined by

$$q_A=-\int\tilde{\rho}_A d\mathbf{r}=\sum_n C_n^A\int\frac{1}{4\pi}\nabla^2 f_n^A d\mathbf{r}. \tag{10}$$

The forgoing formulation of atomic charge can be easily extended to include higher multipoles. By including functions of corresponding higher multipoles in the expansion Eq. (3), it would certainly improve the representation of the electrostatic potential. However, the set of atomic charges and multipoles obtained should be used together and we lose the simple point charge model typically employed in molecular simulations. The main focus of this paper is to find the best possible atomic point charges. To serve this purpose, we should have only the atomic spherical functions in the expansion. We will next demonstrate that the atomic charges obtained this way are in general superior to ESPD charges for the representation of the electrostatic potential.

A related approach was taken by Koster *et al.*[24] to model molecular electrostatic potential. Their focus was on the approximation of the molecular electrostatic potential. First

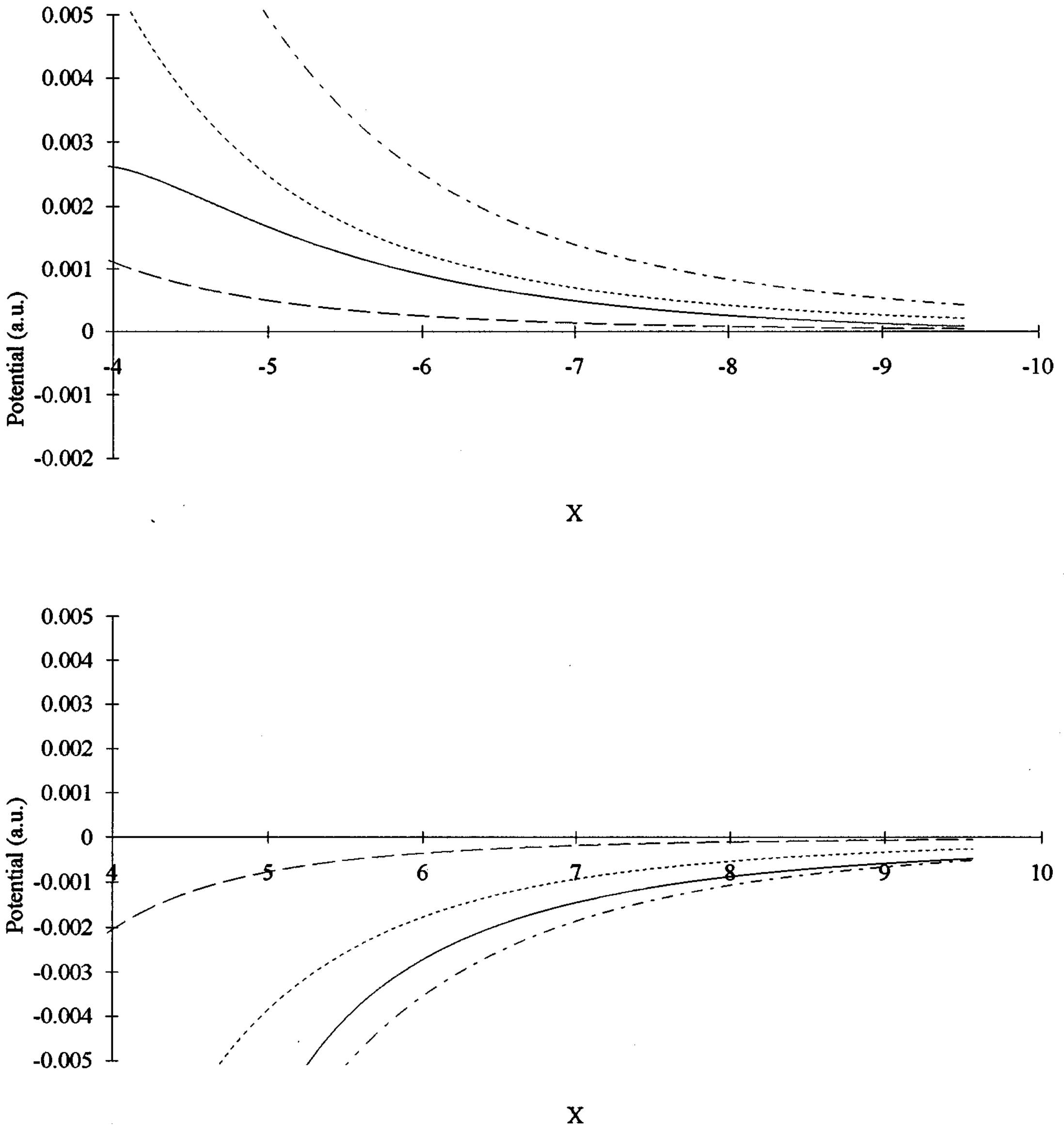


FIG. 2. The potential along the CH bond in the CH_4 molecule. The C atom is located at $X=0.00$ and the H atom is located at $X=1.91$. Solid line, SCF potential. Long dash, potential from point charge model with charge $(C)=-0.1$. Short dash, potential from point charge model with charge $(C)=-0.5$. Dot–dash, potential from point charge model with charge $(C)=-1.0$.

they obtained the atomic multipoles (atomic charges and dipoles) from their multipole moment analysis (MMA), then they determined the approximate electrostatic potential by solving the Poisson equation, Eq. (2), for each atom, so that the potential has the exact asymptotic behavior both at the nuclei and at the large distance. Our approach is different in determining the atomic multipoles (atomic charges here) from solution of the Poisson equation, because the Poisson equation is solved for the entire molecule rather for the constituent atoms.

COMPUTATIONAL METHODOLOGY

In the present work, we used the deformation density as input to Eq. (1), which is the difference between the molecular electron density and the sum of the spherical atomic densities of the free atoms centered at the corresponding nuclear positions. Equations (1) to (9) apply equally to the deformation density and the charges given by Eq. (10) are the net atomic charges. The use of the deformation density facilitates faster convergence of the calculated charges with respect to the number of basis functions used in the expansion in Eq. (3). However, both procedures give essentially the same converged atomic charges (Table II).

The short-range behavior of the electrostatic potential was modeled using Slater-type basis functions [Eq. (11a)], and the long-range behavior was modeled by a "long-range" basis function [Eq. (11b)] with correct $1/r$ asymptotic behavior,

$$f_n^A(r)=N_0^{(n-1)}r^{n-1}e^{-\lambda_A r}, \tag{11a}$$

$$N_0^{(n)}=\sqrt{\frac{(2\times\lambda_A)^{2n+3}}{4\pi(2n+2)!}},\quad n=1\ldots N;$$

$$f_0^A(r)=\frac{1}{r}\,(1-e^{-\lambda_A' r}),\tag{11b}$$

where $N_0^{(n)}$ is the normalization constant and the λ_A and λ_A' are positive parameters. Similar basis functions have been used by Koster *et al.*[24] in the approximation of electrostatic potentials.

For this choice of basis, the atomic charges $\{q_A\}$ are determined by the coefficients of the long-range basis functions,

$$q_A=\sum_n C_n^A\int \frac{1}{4\pi}\,\nabla^2 f_n^A d\mathbf{r}=-C_0^A.\tag{12}$$

The exponents $\{\lambda_A\}$ were determined empirically for each atom type by inputting the spherical atomic density of an isolated atom and optimizing the exponent by calculating the atomic charge on this atom, which should equal to the atomic number. The results are listed in Table I. The exponents do not affect the atomic charges significantly. For example, the exponent of the H atom in the HF molecule can vary from 1.5 to 4.0 with the atomic charge of the H atom changing from 0.4013 to 0.4014. The exponents for the long-range functions $\{\lambda_A'\}$ had little effect on the overall results, and were taken to be twice the corresponding $\{\lambda_A\}$ values.

A total of 15 Slater-type functions were used to model the short-range potential. This was adequate to provide stable convergence of U, and in most cases stable charges (Fig. 1). For exceptional cases such as CH_4, converged charge results cannot be obtained; however, this is not a significant limitation, as will be discussed later.

The matrix elements $(\rho)_{A,n}$ were calculated numerically with three-dimensional multicenter quadrature; the rest of the matrix elements in Eq. (6) were integrated analytically.[25] The electron density and related properties of various test molecules were determined with a self-consistent-field (SCF) program based on the conventional Kohn–Sham density-functional theory[26] with a numerical basis. Basis functions used in the SCF calculations were obtained from numerical solutions of the isolated atoms, similar to that of Delley.[27] A basis set of $3s$, $2p$, and $1d$ functions was used for hydrogen atoms, and a basis set of $4s$, $3p$, and $2d$ functions was used for all other atoms.

RESULTS

Table II compares ESPD charges to those determined from the present method. The ESPD results were obtained by a least-squares fit to the SCF electrostatic potential on the set of four scaled van der Waals surfaces (scaling factors 1.2, 1.4, 1.6, and 1.8) (Ref. 28) at a density of 80 points per square angstrom. Overall, the two methods give similar charges.

It is known that dipole moments calculated by density-functional theory, in most cases, agree more closely with experiment than those obtained from Hartree–Fock calculations.[29] For example, the Hartree–Fock results predict the wrong sign for the dipole moment of the CO molecule, whereas density-functional theory gives the correct sign.[29,30] Table III shows the dipole moments from density-functional theory SCF calculations are very close to experimental results, and the simple charge model can reproduce the dipole moments and give the correct sign for each molecule. In addition, our method gives better agreement with the SCF results than the ESPD charges.

The choice of fitting points above has been employed elsewhere,[17] but is somewhat arbitrary. Table IV demonstrates the dependence of ESPD charges on the choice of fitting points. The present method circumvents the ambiguity associated with choosing a set of fitting points by use of three-dimensional integrations in the construction of the necessary matrix elements.

The present method has the advantage that it not only gives charges, but also determines a spherical potential model representation for each atom. Table V compares the variance of the calculated electrostatic potential from the SCF potential for CO_2 at the fit points described above. In general, the spherical model potential gives the best agreement with the SCF potential, whereas the potential of different point charge models are very similar to one another.

In the case of CH_4, we have obtained a converged value for the functional U, but we cannot obtain stable atomic charges (Fig. 1). This can be explained by the fact that the atomic charge is a long-range phenomena, and with our method the charge is determined by the coefficient of the long-range (asymptotic $1/r$) basis function. The electrostatic potential around CH_4, however, is short-range since it has vanishing monopole, dipole, and quadrupole moments. This is illustrated in Fig. 2. The electrostatic potential for CH_4 is very small in the region outside the van der Waals radius, and is not significantly affected by varying the charge. Thus, the atomic charges do not have much meaning in this case, which is reflected by the large variance in the computed values. Nevertheless, the convergence of the functional U, in spite of the fluctuation in the atomic charge in this case, indicates the convergence of the calculated spherical model potential.

To examine the necessity of using the spherical potential model in place of the point charge model, we examine the interaction energy, defined as the difference between the total energy of the interacting system and the total energy of isolated molecules. The electrostatic interaction energy is defined as the classical electrostatic energy component of the total interaction energy for a system composed of point-charged nuclei and fixed electronic densities.[31] In general, the electrostatic interaction energy is modeled by a simple point charge model.[32] We calculated the electrostatic interaction energy by using the point charge model and the spherical potential model for comparison.

Figure 3 is a typical bimolecular interaction energy curve.[31] For molecular simulations, the electrostatic interaction energy will not be important in the core repulsive region where molecules have small probability of occupying. Figures 4 and 5 show the interaction energy and the electrostatic interaction energy between two HF molecules in different orientations. In the head-on case (Fig. 4), the point charge

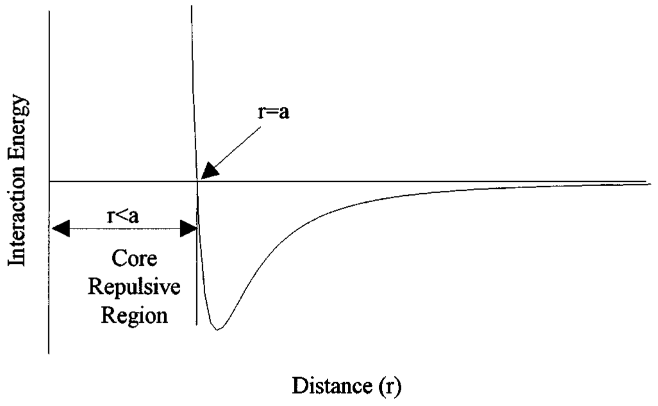


FIG. 3. A typical interaction energy curve for a two-molecule system.

model can describe the electrostatic interaction energy well in the range $r > 4.3$ a.u. outside the core repulsive region. Thus the point charge model is acceptable in this case.

In the "stacked" case (Fig. 5), however, the point charge model poorly represents the electrostatic interaction energy outside the core repulsive region ($r > 3.7$ a.u.). In the range of $r = 3.7–5.5$ a.u., the spherical potential model gives significantly better results.

DISCUSSION

We have presented a new method for determining atomic charges in molecules. The results show the charges obtained from this method are similar to ESPD charges that are widely used for determining parameters in molecular simulations. Unlike the electrostatic potential fitting procedures, the method described here does not rely on arbitrary fitting points. Hence, this method is unique, and can be used as an alternative to obtain atomic charges for molecular simulations. This method has the additional advantage that it depends only on the electron density. Since the electron density is an observable quantity and can be obtained by experiments, this method can be used to calculate charges directly from experimental data.

It has been shown that certain methods based on population analysis or partitioning of the molecular density result in accurate molecular dipole moments, but require atomic dipole or multipole contributions.[16,33] The present method is designed to give the best possible spherical-atom representation of the electric field, and has been demonstrated to give accurate molecular dipole moments without requiring higher order multipole terms. As a further demonstration of the quality of the spherical atom model potential, we have included higher order angular momentum p-type functions in Eq. (3), and repeated calculations for the CO molecule. In-

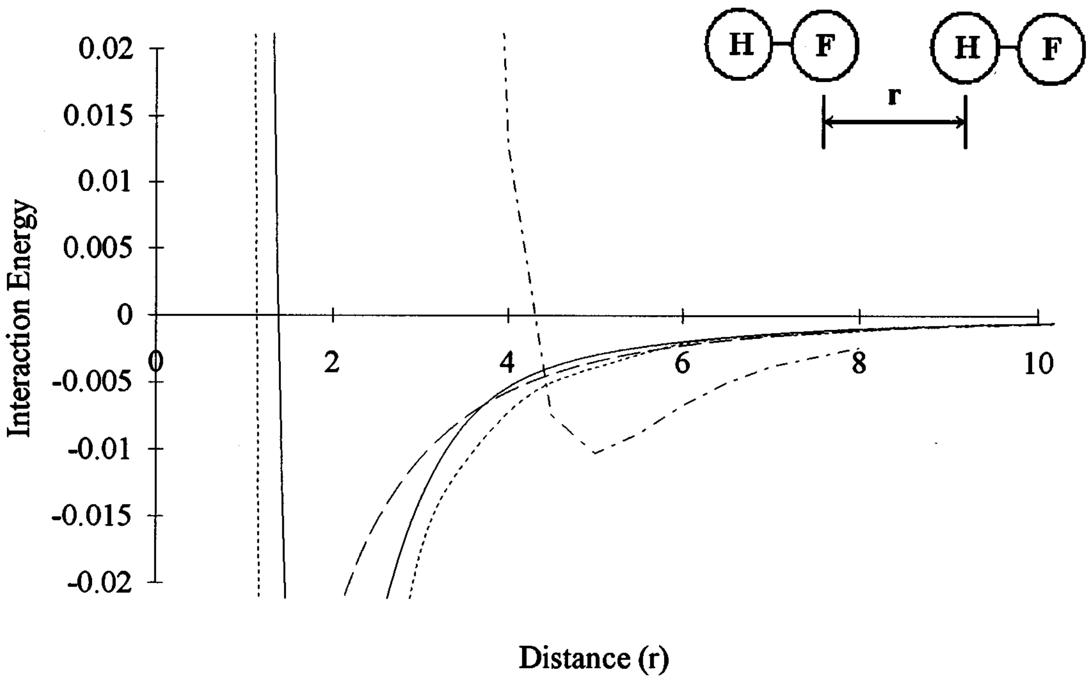


FIG. 4. The comparison of interaction energy curves for head-on HF case. Solid line, electrostatic interaction energy from the SCF density. Short dash, electrostatic interaction energy from the ESPD point charge model. Long dash, electrostatic interaction energy from the spherical potential model. Dot–dash, total interaction energy from SCF calculations. The total interaction energy curve indicates the range of $r > 4.3$ a.u. is important for simulations. In this range the point charge model is good enough to reproduce the electrostatic potential. All units are atomic units.

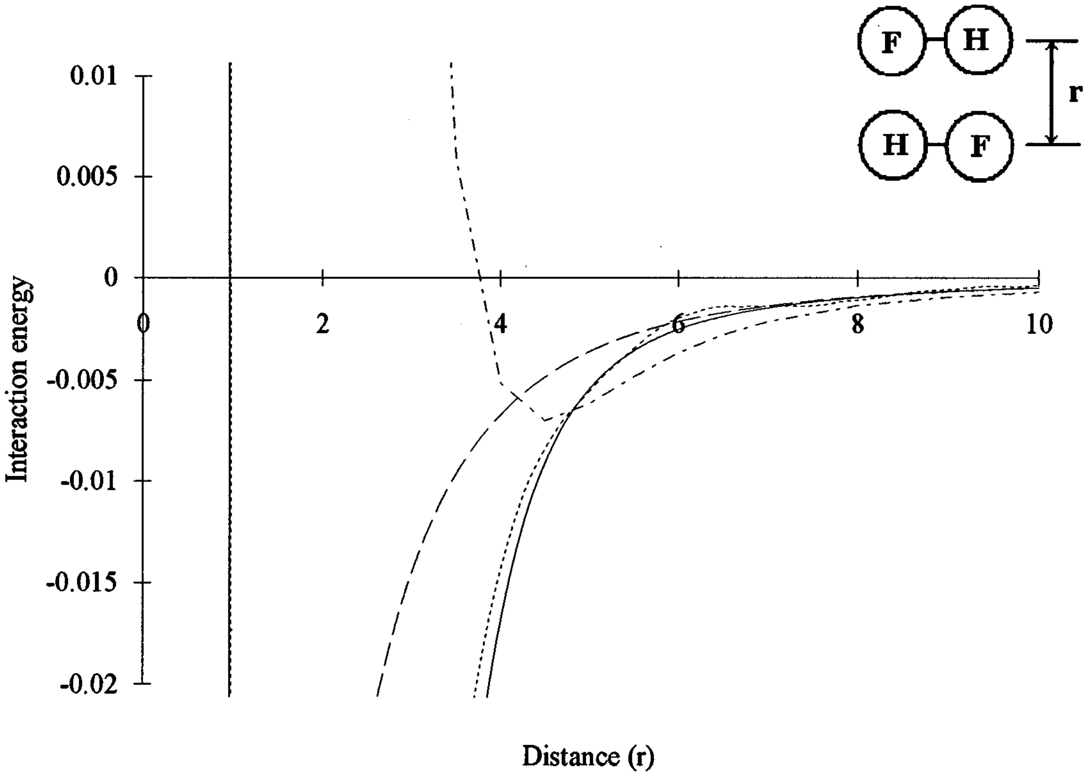


FIG. 5. The comparison of interaction energy curves for stacked HF case. Solid line, electrostatic interaction energy from SCF density. Long dash, electrostatic interaction energy from ESPD point charge model. Short dash, electrostatic interaction energy from spherical potential model. Dot–dash, total interaction energy from SCF calculations. The total interaction energy curve indicates the range of $r>3.7$ a.u. is important for simulations but in the range of $r>3.7$ a.u. and $r<5.5$ a.u., the ESPD point charge model is not enough to reproduce the electrostatic potential. All units are atomic units.

clusion of p-type functions does not significantly change the atomic charges. The spherical model and spherical model with p-type function give very similar atomic charges (-0.0473 and -0.0472 for C, respectively) and molecular dipole moments (0.1007 a.u. and 0.1006 a.u., respectively). In the present method the spherical component is highly dominant with little contribution from the atomic dipoles (4.0×10^{-4} a.u. for C and -4.4×10^{-4} a.u. for O). Thus the atomic charges and other higher order moments derived from our model should be considered as a set of empirical parameters designed to best reproduce the electrostatic potential and they may not be suitable for other interpretations.

Finally, the method presented provides a prescription for determining spherical-atom potential models for atoms in molecules. In some cases, the point charge model is inadequate to describe the interaction energy between molecules. The description of the interaction energy is much improved when the spherical atom potential models are used (e.g., stacked HF case in Fig. 5). Such models may prove useful in future generation force fields that abandon the point charge description of atoms. Moreover, the method is not restricted to spherical-atom models, but can be generalized to other functional forms to include atomic polarization effects. When nonspherical functions are used, one will obtain a more accurate description of the electrostatic potentials but more parameters beside atomic charges will be needed.

ACKNOWLEDGMENTS

T.L. is a Cray Research Fellow. D.M.Y. is a NSF postdoctoral fellow jointly funded by the North Carolina Supercomputing Center, MCNC, RTP, NC. This work has been supported by the NIH Structure Biology Research Resource at the Microelectronics Center of North Carolina, the North Carolina Supercomputing Center, and the National Science Foundation.

[1] K. Jug and Z. B. Maksic, in *Theoretical Model of Chemical Bonding*, edited by Z. B. Maksic (Springer, Berlin, 1991), Part 3, p. 233.
[2] S. Fliszar, *Charge Distributions and Chemical Effects* (Springer, New York, 1983).
[3] P. E. Smith, J. Am. Chem. Soc. **113**, 6029 (1991).
[4] J. Gao, J. Chem. Phys. **98**, 1975 (1993).
[5] P. Cieplak, J. Comp. Chem. **12**, 1232 (1991).
[6] R. S. Mulliken, J. Chem. Phys. **36**, 3428 (1962).
[7] R. S. Mulliken and P. Politzer, J. Chem. Phys. **55**, 5135 (1971).
[8] A. E. Reed, R. B. Weinstock, and F. A. Weinhold, J. Chem. Phys. **83**, 735 (1985).
[9] K. Jug, Theor. Chim. Acta **31**, 63 (1973).
[10] F. L. Hirshfeld, Theor. Chim. Acta **44**, 129 (1977).
[11] R. F. W. Bader, Acc. Chem. Res. **18**, 9 (1985).
[12] R. S. Evans and J. E. Hubeey, Chem. Phys. Lett. **19**, 114 (1973).
[13] L. E. Chirlian and M. M. Francl, J. Comp. Chem. **8**, 894 (1987).
[14] S. R. Cox and D. E. Williams, J. Comp. Chem. **2**, 304 (1981).
[15] J. Cioslowski. Phys. Rev. Lett. **62**, 1469 (1989).

[16] K. B. Wiberg and P. R. Rablen, J. Comp. Chem. **14**, 1504 (1993).

[17] C. I. Bayly, P. Cieplak, W. D. Cornell, and P. A. Kollman, J. Phys. Chem. **97**, 10 269 (1993).

[18] U. C. Singh and P. A. Kollman, J. Comp. Chem. **5**, 129 (1984).

[19] C. Lanczos, *The Variational Principles of Mechanics* (Dover, New York, 1970).

[20] R. G. Parr and W. Yang, *Density Functional Theory of Atoms and Molecules* (Oxford, New York, 1989).

[21] A. Luty, E. Davis, and J. A. McCammon, J. Comp. Chem. **13**, 1114 (1992).

[22] G. G. Hall and D. Martin, Isr. J. Chem. **19**, 255 (1980).

[23] C. M. Smith and G. G. Hall, J. Quantum Chem. **31**, 685 (1987).

[24] A. M. Koster, C. Kolle, and K. Jug, J. Chem. Phys. **99**, 1224 (1993).

[25] J. A. Pople and D. L. Beveridge, *Approximate Molecular Orbital Theory* (McGraw–Hill, New York, 1970).

[26] W. Yang, J. Chem. Phys. **94**, 1208 (1991).

[27] B. Delley, J. Chem. Phys. **92**, 508 (1990).

[28] M. Orozco and F. J. Luque, J. Comp. Chem. **11**, 909 (1990).

[29] L. Fan and T. Ziegler, J. Chem. Phys. **94**, 6057 (1991).

[30] G. E. Scuseria, M. D. Miller, F. Jensen, and J. Geertsen, J. Chem. Phys. **94**, 6660 (1991).

[31] I. G. Kaplan, *Theory of Molecular Interactions* (Elsevier, New York, 1986).

[32] J. A. McCammon and S. C. Harvey, *Dynamics of Proteins and Nucleic Acid* (Cambridge University, Cambridge, 1987), p. 40.

[33] K. Jug, Theor. Chim. Acta **39**, 301 (1975).